Being able to think and reason proportionally is an essential element for understanding and applying mathematics in more complex ways.

Proportional reasoning

Why is proportional reasoning an important notion?

Being able to think and reason proportionally is an essential element for understanding and applying mathematics in more complex ways. People use proportional reasoning to calculate best buys; to perform measurement or monetary currency conversions; to adjust recipes; or to work with drawings and plans.

'Susan Lamon estimates that over 90% of students who enter high school cannot reason well enough to learn mathematics and science with understanding and are unprepared for real applications in statistics, biology, geography or physics. While students may be able to solve a proportion problem with a memorised procedure, this does not mean that they can think proportionally.' (Ontario Ministry of Education, 2012, p. 4)

This paper provides guidelines on proportional reasoning and how leaders can support educators in developing proportional reasoning, both in themselves and in their learners.

What is proportional reasoning?

The essence of proportional reasoning is the consideration of the relationship between two quantities. Learners are using proportional reasoning when they understand that a group of children that grows from 3 to 9 is a more significant change than a group of children that grows from 100 to 150, since the number tripled in the first example, but only grew by 50% in the second example (and thus was not even doubled).

'Proportional reasoning is difficult to define. It is not something that you either can or cannot do but is developed over time through reasoning…It is the ability to think about and compare multiplicative relationships between quantities.' (Van de Walle, 2006, p.154)

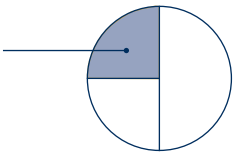
Proportional reasoning involves learners thinking multiplicatively, rather than just additively, about the numbers involved (Behr, Harel, Post & Lesh, 1992). For example, instead of just describing a number or quantity as smaller than or bigger than, learners use terms such as double, half or three-times greater. For example, increasing the quantity of ingredients in a recipe requires that learners keep the ingredients in the right proportions to each other. So, for a recipe that requires 2 cups of flour and 1 cup of sugar; if you add one extra cup of flour you do not also add an extra cup of sugar but rather 1/2 cup, to maintain the correct ratio between ingredients.

Learners use proportional reasoning in early maths learning, for example, when they think of 8 as two fours or four twos rather than thinking of it as four more than four. They use proportional reasoning later in learning when they think of how a speed of 50 km an hour is the same as a speed of 25 km in 30 minutes. Learners continue to use proportional reasoning when they think about slopes of lines and rates of change.

Proportional reasoning also underpins understanding of fractions. For example, when examining fractions, learners need to understand that the number of parts, equal or otherwise, is important, but only in relation to:

- what is defined as 'the whole'
- the size of the part in relation to the whole.

If this circle represents 'the whole', then this segment is not 1/3, even though there are 3 parts: it is 1/4 of the whole.



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How can educators help learners develop proportional reasoning?

The development of proportional reasoning is something that takes time. It is fostered by quality learning experiences in which learners have opportunities to explore, discuss and experiment with proportion situations. For instance students can explore comparison problems (where two ratios might be given for comparison) involving contexts such as cordial strength or paint colour density, before working with more complex missing value problems (Siemon et al, 2015). Proportional reasoning is also dependent upon sound foundations of multiplication and division.

Learners may develop proportional reasoning when:

- fractions are taught holistically and in real-world contexts (Lesh, Post & Behr, 1988)
- questions build on their intuitive understanding and confront their own misconceptions
- concrete materials are used for modelling where appropriate
- educators focus on the multiplicative relationships involved, particularly when questions involve increasing or decreasing quantities. For example, for a cordial mixture which is one part syrup and 4 parts water; if we add 5 ml of syrup we must add 20 ml of water to maintain the strength of the mixture.

Learners naturally tend to consider change situations in additive terms, but when prompted through questioning they can develop understanding in both additive and multiplicative terms.

For example, consider the following situation:

Ruby and Trent have different amounts of tokens.

Examples of questions requiring additive thinking:

- **Who has more tokens, Ruby or Trent?**
- **How many more tokens does Trent have than Ruby?**
- **How many fewer tokens does Ruby have than Trent?**

Examples of questions requiring multiplicative thinking:

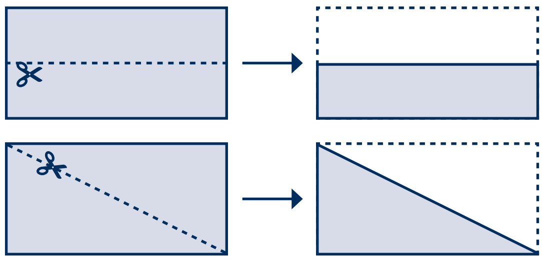
- **How many times would you have to stack Ruby's tokens to get a pile as high as Trent's?**
- **What part of a dozen tokens does Ruby have?**
- **What part of a dozen tokens does Trent have?**

How can leaders support their staff?

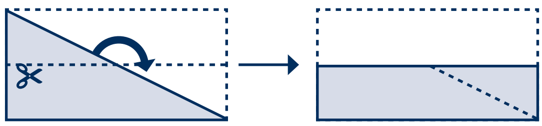
Leaders can support their teams of educators to develop their own understanding of proportionality by encouraging them to take the role of learners in problem-solving tasks and to do the thinking to confront their own misconceptions. Lamon (2012) estimates that well over 90% of adults do not reason proportionally. This implies that many of the educators working in our schools will not have developed this conceptual understanding for themselves, including confidence in working with fractions, decimals, percentages, rates and ratios. One starting point would be to use some of the tasks suggested below to uncover staff misconceptions.

For example:

- **Take a few sheets of rectangular paper (of the same size). Cut them into halves in as many ways as possible, for example:**



Test to make sure these 'halves' are equivalent.



- **Using a recipe designed to feed 4 people, adjust it to feed 10 people. Next, adjust it to feed 3 people.**
- **Given a selection of shapes or objects, find the proportion of: red to blue; triangles to quadrilaterals; round shapes to flat shapes…**
- **Using the speed of a car and a known distance to travel, calculate how long it will take to get to a destination. How does this change when you speed up or slow down?**
- **Add $\frac{1}{3}$ and $\frac{1}{5}$ by drawing it.**
- **Find $\frac{2}{3}$ of 4.**

When observing lessons, look for learners who:

- are engaged in problem-solving tasks designed to confront common misconceptions
- can identify multiplicative relationships and work with the units involved
- are prepared to question their own misconceptions when faced with challenging problems
- use concrete materials to explore proportionality.

Leaders should consider how they work with their staff to incorporate the big ideas in number into common agreements around planning, teaching and assessment at their site.

Reflective questions for leaders to ask their teachers

When looking at and discussing the numeracy and mathematics program, you could, for example, ask the teacher:

- How have you set up opportunities for students to think about and solve real life problems using proportional reasoning skills?
- What evidence do you have to show some students are using proportional reasoning?
- Have you used the proportional reasoning common misunderstanding tools? (Siemon, 2009)

Further resources

The big ideas in number are discussed in further detail in the following mathematics papers:

- 3.0 Conceptual understanding: Number and algebra
- 3.1 Trusting the count
- 3.2 Place value
- 3.3 Multiplicative thinking
- 3.4 Partitioning
- 3.6 Generalising.

All papers in this series are based on the work of Dianne Siemon, Professor of Mathematics Education at RMIT and a key text (Siemon et al, 2015).

http://bit.ly/BestAdviceSeries

Further reading

AAMT ‘Proportional reasoning’, Top Drawer Teachers: resources for teachers of mathematics: http://topdrawer.aamt.edu.au/Reasoning/Big-ideas/If-then/Proportional-reasoning (accessed January 2017)

ACER Concept builder: Proportional reasoning, available at https://www.acer.edu.au/pat/pat-teaching-resources-centre/sample-content/proportional-reasoning (accessed January 2017)

Dole S, *Proportional reasoning*, retrieved from http://www.proportionalreasoning.com/index.html (accessed February 2017). Houses resources, activities, research projects and papers for proportional reasoning.

Kennedy T (2015) *Fixing misconceptions in fractions: interventions in mathematics*, Townsville, Queensland: Kennedy Press Pty Ltd

Van De Walle JA, Karp K & Bay-Williams JM (2016) *Elementary and Middle School Mathematics: Teaching Developmentally*, Ninth global edition, UK: Pearson Education Limited. In particular, refer to chapter on ‘Algebraic thinking: Generalisations, patterns and functions’.

Victorian Department of Education and Training, Mathematics Developmental Continuum F–10 This resource provides evidence-based indicators of progress, linked to powerful teaching strategies.

Victorian Department of Education and Training, Assessment for Common Misunderstandings These tools draw on highly focussed, research-based Probe Tasks and the Probe Task Manual (RMIT), as well as a number of additional tasks and resources which have been organised to address ‘common misunderstandings’.

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Lamon S (2012) *Teaching fractions and ratios for understanding: essential content knowledge and instructional strategies for teachers*, (3rd ed.), Routledge

Lesh R, Post T & Behr M (1988) ‘Proportional reasoning’, in J Hiebert & M Behr (Eds.), *Number concepts and operations in the middle grades* (Vol. 2, pp. 93–118), Reston, Virginia: Lawrence Erlbaum

Ontario Ministry of Education (2012) *Paying attention to proportional reasoning: Support document for paying attention to mathematical education*, Queen’s Printer for Ontario, retrieved from http://www.edu.gov.on.ca/eng/teachers/studentsuccess/ProportionReason.pdf

Siemon D (2009) *Proportional reasoning tools: Common misunderstanding*, Department of Education and Early Childhood Development, State of Victoria, available from https://edi.sa.edu.au/library/document-library/learning-improvement/strategic-design/di-siemon-diagnostic-tools/5-PROPORTIONAL-REASONING-DIAGNOSTIC.pdf

Siemon D, Beswick K, Brady K, Clark J, Faragher R & Warren E (2015) *Teaching Mathematics: Foundations to Middle Years*, 2nd edition, Melbourne, Oxford University Press

Van de Walle J & Lovin LA (2006) *Teaching student-centered mathematics: Grades 5–8*, Boston, MA: Pearson, Allyn & Bacon

This paper is part of the DECD Leading Learning Improvement *Best advice* series, which aims to provide leaders with the research and resource tools to lead learning improvement across learning areas within their site.

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